Numerical optimization in cable railway planning

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Planning a cable railway is a complex task. One has to take into account many aspects and an optimal solution is not well defined. The calculation of the cable configurations for given support positions, cable pretension and cable types however is rather formalizable and presents a direct problem. In this research work we study the first steps to solve the inverse problem: computation of optimal support positions for a given support cable type and cable car mass. We define an appropriate cost functional (objective function) with several constraints and use numerical minimization strategies to obtain optimal solutions. Computations of cable configurations are performed, as required by the norms.

Keywords: Cable Railway, Optimization, Numerical Cable Support Computation

I. INTRODUCTION

The simulation of technical systems is typically a direct problem. The evolution equations for the system (usually differential equations) and the initial/boundary conditions are known.

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An appropriate solver gives us the (unique) state of the system at a later time. For the field of cable ropeways the solver consists of a program, which allows the computation of the various cable configurations, when the cable car is moving, the forces on the supports etc. [1–3]. The equations of motion of classical mechanics and elasticity theory are the basis for this task [4]. Model parameters such as support positions, cable types, etc. usually are selected by the engineer and are input quantities.

A new, large research field for applied mathematics as well as engineering and physics is opening when inverse problems are considered. Inverse problems are typical for example in scattering theory, where from the incident and scattered waves one has to deduce the scattering potential (e.g. in applications of nuclear physics or nondestructive testing of steel by ultrasound [5, 6]). In the context of a cable railway inverse problems arise when one looks for an algorithm, which determines the model parameters autonomously [7]. For example, where are the best positions of the supports for a ropeway when a mountain profile is given? Or which cable type is optimal and how much one should strain? There are many questions, and the first of all is how we define a good solution and distinguish it from a bad solution. The overview in Fig. 1 gives an impression of the several planning steps of a cable railway. The choice of the cable path is influenced by many requirements and norms, as well as restrictions. In this work we address the particular problem to find the optimal support positions for a cable railway. We define an appropriate cost functional and constraints. We use global and local optimization techniques to identify the optimal coordinates of a given constant number of supports. For this purpose we use two different models. When one searches a local minimum of a function, then a relatively small number of function evaluations are sufficient to find the solution. In this case we can use a sophisticated program to compute all quantities needed to evaluate the cost functional and the constraints. For each iteration step in the minimization procedure we are able to evaluate a complete ride of the cable car from valley to the mountain station (which takes less than a minute on a PC). Global optimization requires much more computational effort. To obtain a tractable model, we introduce suitable approximations for the determination of the cable car trajectory. In particular, we assume that the support cable is spanned with a counter weight in the valley station. This assumption drastically simplifies the computation of the support cable configurations [8]. Further simplifications were done:

- The movement of the cable car is assumed quasi–static (corresponding to the norm

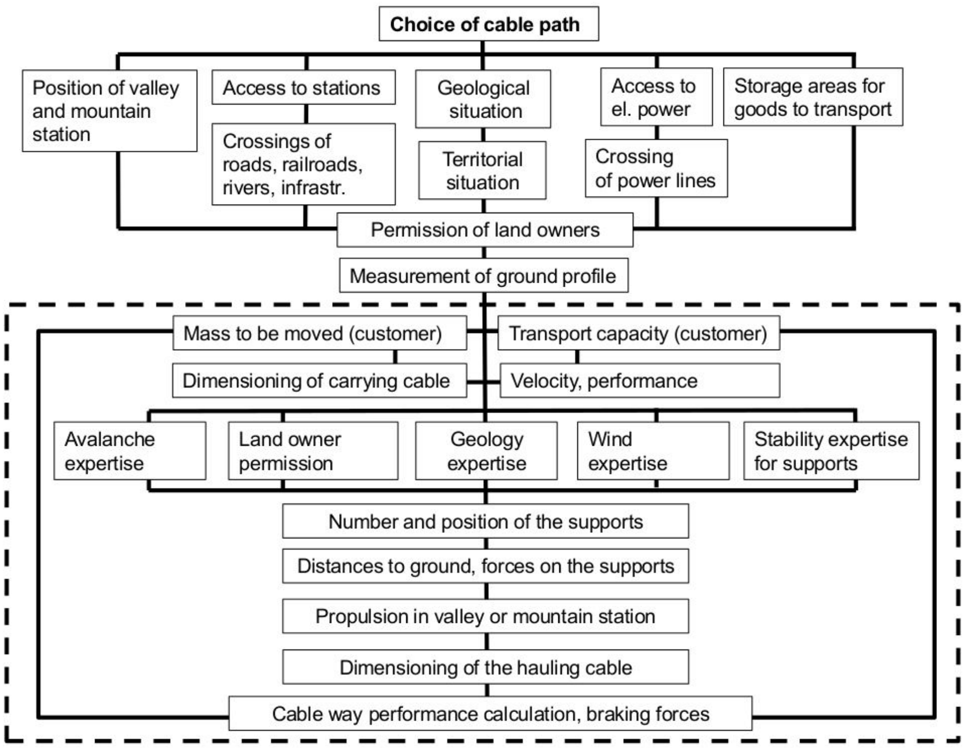

FIG. 1: Planning of a cable railway: aspects of the planning steps.

[9]).

- We neglect temperature effects. We work with one reference temperature only.
- We neglect the problem of the hauling cable (in particular its distance from the ground).
- Important questions about cable oscillations and their minimization are ignored [10].

II. APPROXIMATE CABLE CURVE AND CABLE CAR TRAJECTORY

The function to describe an inelastic cable in a gravitational field, the catenary, is well known [11]. For cable railway applications the elasticity must be taken into account, for small strains the "elastic catenary" in a $x - y$ coordinate system is given [12] by

$$y = \bar{a} \cosh\left(\frac{x - \bar{x}_m}{\bar{a}}\right) - \bar{c} - \frac{1}{2}\frac{g\rho_L}{\mathcal{E}A}\bar{a}^2 \sinh^2\left(\frac{x - \bar{x}_m}{\bar{a}}\right) \ . \quad (1)$$

In this expression g is the earth acceleration, ρ_L the linear mass density (kg/m), $\mathcal{E}$ the isothermal modulus of elasticity and A the cross section of the cable, respectively. The three parameters $\bar{a}, \bar{x}_m, \bar{c}$ determine the cable curve uniquely and are fixed by appropriate boundary conditions. From Eq. (1) one deduces a parabola approximation for the empty cable [8]. Apart from this approximation the determination of the point load trajectory depends on the boundaries of the support cable. If the support cable is spanned with a weight, the problem is statically determined. In the other case, where the cable is held fixed at the end points, the problem is statically undetermined and a solution requires principles from elasticity theory [13]. The numerical computation of the cable configurations and the catenary sag of the point load, the forces on the supports etc. in this case is very time–consuming, which makes it difficult to use global optimization. So, in order to obtain results we also use a crude approximation for the computation of the point load trajectory and the empty cable configuration (see Fig. 2).

Let T_0 be the gravitational force of the counterweight, and let m be the mass of the point load. We consider one span only with cartesian coordinates of the supports X_1, Y_1 and X_2, Y_2. The mid – span catenary sag of a point–loaded cable is given [16] by

$$f_m = \frac{2mg + \rho_L g l}{(T_0 + \rho_L g y_m)\cos(\gamma)} \frac{l}{8} \ , \tag{2}$$

where we use the abbreviations

$$l = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2} \ , \quad \gamma = \arctan \Delta \ , \quad \Delta \equiv \frac{Y_2 - Y_1}{X_2 - X_1} \ . \tag{3}$$

l denotes the length and γ the slope angle of the chord respectively. $y_m = (Y_1 + Y_2)/2$ is the mean $y-$ value of the span. Given the coordinates of both supports and f_m, one can compute the trajectory of the point load using a parabola approximation (see the lower part of Fig. 6). We obtain for $X_1 \leq x \leq X_2$,

$$f_2(X_1, Y_1; X_2, Y_2; x) = ax^2 + bx + c \ , \tag{4}$$

with

$$a = \frac{X_1(Y_m - Y_2) + X_2(Y_1 - Y_m) + X_m(Y_2 - Y_1)}{(X_1 - X_2)(X_1 - X_m)(X_2 - X_m)} \ , \tag{5}$$

$$b = \frac{X_m^2(Y_1 - Y_2) + X_1^2(Y_2 - Y_m) + X_2^2(Y_m - Y_1)}{(X_1 - X_2)(X_1 - X_m)(X_2 - X_m)} \ , \tag{6}$$

$$c = \frac{X_m[X_2(X_2 - X_m)Y_1 + X_1(X_m - X_1)Y_2] + X_1(X_1 - X_2)X_2 Y_m}{(X_1 - X_2)(X_1 - X_m)(X_2 - X_m)} \ . \tag{7}$$

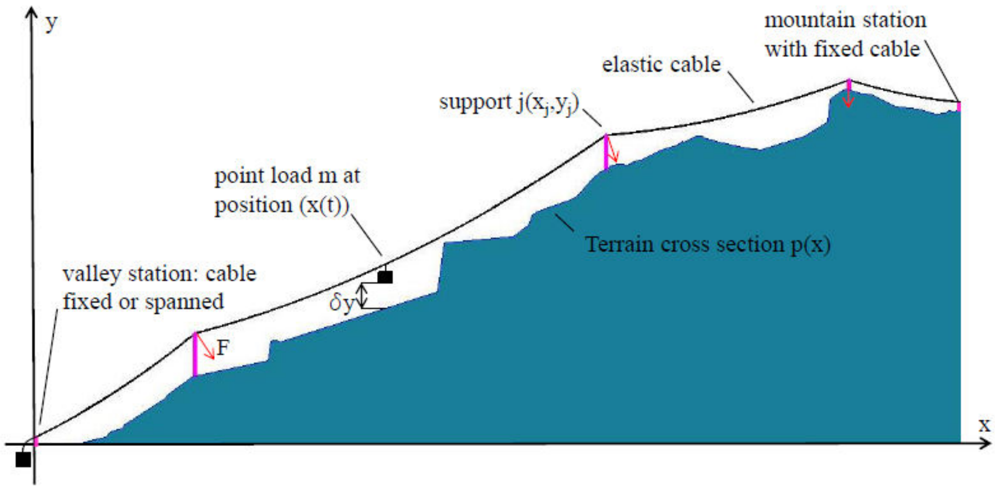

FIG. 2: Mechanical model used in our numerical simulations.

Here we have defined:

$$X_m = \frac{X_1 + X_2}{2} \ , \tag{8}$$

$$Y_m = \Delta \cdot (X_m - X_2) + Y_2 - f_m \ . \tag{9}$$

With these expressions the trace of the point load trajectory $(x, f(x))$ as well as the empty cable curve (obtained by setting $m = 0$ in Eq. (2)) are computable for an arbitrary number of supports. Using the step function $\Theta(x) = 1$, $x > 0$, $\Theta(x) = 0$, $x < 0$, one can write

$$f(x) = \sum_j f_2(X_j, Y_j; X_{j+1}, Y_{j+1}; x)\Theta(x - X_j)\Theta(-(x - X_{j+1})). \tag{10}$$

The sum runs over all support coordinates including the valley and mountain stations. All considerations are based on a quasi–static movement of the point load, so the time–dependent trajectory is $(x_{\rm pl}(t), f(x_{\rm pl}(t))$, where $x_{\rm pl}(t)$ is the time–dependent $x-$ coordinate of the point load.

III. SETUP OF THE OPTIMIZATION STRATEGY

The optimization strategy requires in general the definition of a cost functional which has to be minimized and several constraints. In our case, the definition of an appropriate cost functional J is a non–trivial task. More than one desirable target makes the selection of the best cost functional ambiguous (multiobjective optimization). It is important however to keep in mind that a mathematical algorithm should help to reduce the number of possible solutions (reduction of the dimension of the search space). Otherwise the algorithm is valueless. Here we use a minimal support height for a fixed support number, in order to increase stability against crippling. Questions about robustness of the obtained solutions are ignored at the moment [14]. When one searches the minimum of a function, the choice of the initial guess is an important question. When we use local optimization, we construct the starting points using the following strategy: The terrain profile given by satellite measurements is interpolated linearly. Start- and end position of the cableway coincide with the first and the last pair of coordinates. Starting with a middle height of the cable supports (averaged between a reasonable minimal and maximal value) a first algorithm looks for further necessary supports between the existing supports, using chord junctions and evaluating positions in the interpolated profile with too little distance from the chord or even situated above the chord choosing the position with the most critical distance d_{crit} first. Here d_{crit} is defined trough Eq. (12) with $T_0 \to \infty$ and $m = 0$ and is the first absolute minimum counted from the left.

Considering a necessary minimal distance between the empty cable and the profile of the terrain and further the sag of the chosen cable due to its weight, elasticity and tension a second algorithm looks for further necessary support points choosing the position with the most critical distance first putting there the next support.

A third algorithm simulates the movement of the cable car at full weight along the support cable considering a maximum of support height and cable tension in the valley station to look for further necessary supports. Afterwards the optimization process starts using the chosen support positions, the maximum of height for each support.

Global optimization strategies usually does not require initial guesses but a closed region, which contains probably the optimal solution [17].

A. Cost functional

First we define the mountain profile as a real function $p(x)$, where $x \in I$, and $I = [0, X_{max}]$ is some interval [18]. We use the cost functional

$$J = \sum_{j=1}^{N} h_j \ , \tag{11}$$

where $h_j \equiv Y_j - p(X_j)$ is the height of the j-th support. The coordinates X_j, Y_j are variational parameters, N is the number of supports. As first numerical studies have shown, this approach reflects an important part of what in reality is useful. In fact increase the critical crippling force with the height of the support, so it is advantageous to use supports so low as possible.

B. Constraints

The optimal solution must fulfill several constraints. We denote these by C_0, C_1.... Constraints can be defined trough equalities or inequalities. Here the introduced constraints are written either as equalities $C_j = 0$ or inequalities $C_j \leq 0$ respectively.
The most important constraint is the distance "car–ground" δy_{min}, which must be greater than a certain value (e.g. $\delta y_{min} = 5$ m) to avoid car–ground collisions. The first constraint C_0 consequently reads

$$C_0 = \delta y_{min} - \min_{x_{\mathrm{pl}} \in I} \left[f(x_{\mathrm{pl}}) - p(x_{\mathrm{pl}}) \right] \ , \tag{12}$$

where x_{pl} is the point load position. The second constraint ensures that the support cable during the raid of the car does not leave the support [for a discussion of this point see [8, 9]]. The cable–support force at the support no. j is denoted as [19]

$$\mathbf{F}^j(x_{\mathrm{pl}}) = F_x^j(x_{\mathrm{pl}})\mathbf{e}_x + F_y^j(x_{\mathrm{pl}})\mathbf{e}_y \ . \tag{13}$$

The components $F_x^j(x_{\mathrm{pl}})$, $F_y^j(x_{\mathrm{pl}})$ obviously depend on the position of the point load. We are interested for the vertical component $F_y^j(x_{\mathrm{pl}})$, which must be at least negative for all supports and all positions x_{pl}. Let us define the boundary value for $F_y^j(x_{\mathrm{pl}})$ with $F_{y,\min}^j$. So we have

$$F_y^j(x_{\mathrm{pl}}) \leq F_{y,\min}^j \quad \forall j \ , x_{\mathrm{pl}}. \tag{14}$$

In realistic situations the values $F^j_{y,\text{min}}$ are not constant but depend on the positions of the supports. This is due to the wind–induced vertical forces, which must be taken into account when computing $F^j_{y,\text{min}}$ [15]. At the moment we neglect this fact and use a constant value for all supports. The constraints, corresponding to Eq. (14), are

$$C_j = \max_{x_{\text{pl}} \in I} \left[F^j_y(x_{\text{pl}}) - F^j_{y,\text{min}} \right] \ , \qquad j = 1, 2, ...N \tag{15}$$

For a support cable fixed at the ends of the line, the cable tension depends on the position of the point load and in particular the determination of the maximal cable tension T_{max} requires some computational effort. On the other hand, the situation is quite simple when one end of the cable is movable. If one neglects friction resistances on the support, the maximal cable tension is constant i.e. does not depend on the position of the point load. In any case the maximal cable tension must be in the safe range regarding the minimal breaking tension T_{br}. This yields the constraint

$$C_{N+1} = \nu T_{\text{max}} - T_{\text{br}} \ . \tag{16}$$

Usually one sets for the security factor $\nu = 3.5$ for person transport ropeways.

IV. NUMERICAL EXAMPLES

In this section we present two numerical examples to demonstrate the theory given in the previous sections.

A. Case study: linear profile

A linear mountain profile, (horizontal 1000 m, vertical 500 m, Fig. 3), is the simplest possible case to check the optimization strategy and the numerical results. We use the *"Differential Evolution"* Algorithm to minimize Eq. (11) with constraint $C_0 = 0$. As Fig. 3 shows, the position of the support takes place in the first half of the profile ($X < 500$ m). This is reasonable, because the cable tension grows with increasing height. So the first span has a greater mid–span catenary sag than the second one. The end–points of the line are held fixed at a height of 5 m. Fig. 4 shows a similar result for a support cable fixed at both ends of the line. A global minimum for the support height exists, which can be found

bound. cond.	T_0 [kN]	100	150	200
end fix	X_{min} [m]	475	482.2	484.7
end fix	h_{min} [m]	63.3	49.1	40.3
sp. weight	X_{min} [m]	419.7	446.9	460.2
sp. weight	h_{min} [m]	105.5	72.2	55.6

TABLE I: Optimal support positions and heights for a cable fixed at the end points and spanned with a counterweight T_0. Parameters as in Fig. 3.

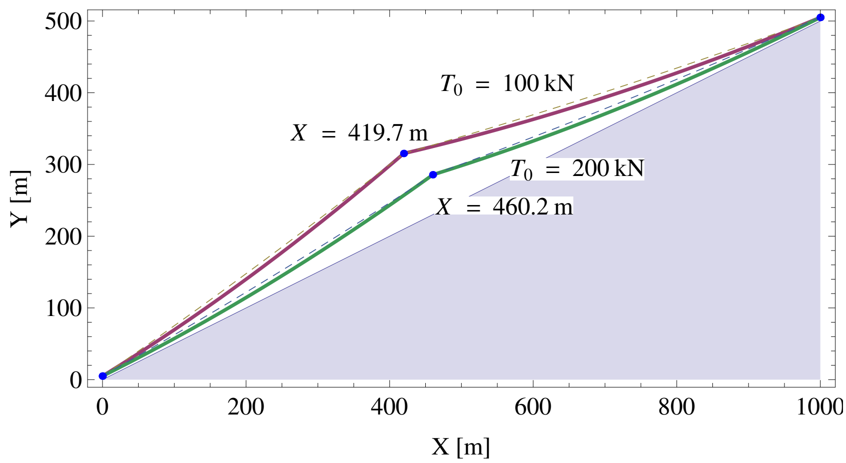

FIG. 3: Results for a linear profile (slope 1/2) with 1 support. Two cases with different spanning weight T_0 are shown: For $T_0 = 100$ kN one obtains $J \equiv h_1 = 105.5$ m and for $T_0 = 200$ kN $J = h_1 = 55.6$ m ($\delta y_{min} = 5$ m). Dashed lines indicate the empty cables. Cable parameters here and in all other figures: a locked coil rope with diameter 21 mm and a metallic cross section of 301 mm^2 was used. The linear mass density is $\rho_L = 2.53$ kg/m, the breaking tension $T_{\text{br}} = 530$ kN, the mass of the point load is $m = 1000$ kg respectively.

using standard local optimization techniques such as *"Line search"* methods. Tab. I shows a summary of results for both types of boundary conditions: cable fixed at the end points and spanned with a counterweight. As expected, for a cable fixed at the endpoints, the supports are quite lower.

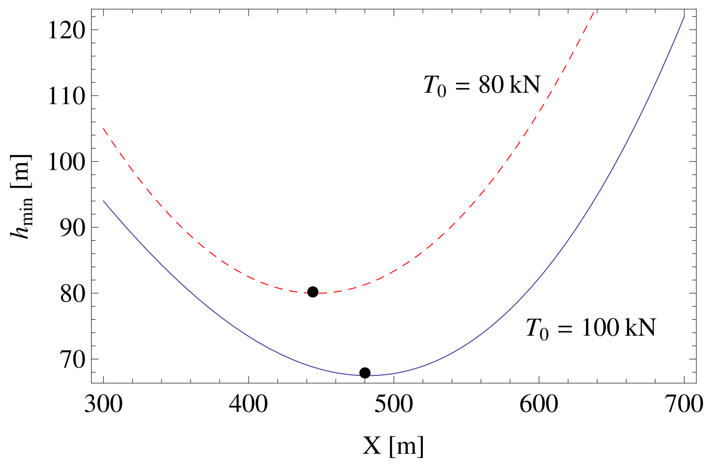

FIG. 4: Plot of the minimal support height versus the support position for the linear profile and support cable fixed at the end points. The only constraint considered in this example is $C_0 = 0$, with $\delta y_{min} = 5$ m. The red, dashed line corresponds to $T_0 = 80$ kN, the solid, blue line to $T_0 = 100$ kN respectively. The global minimum of J in the latter case is $h_{min} \approx 68$ m at $X \approx 480$ m. As expectable the position of the minimum goes versus the middle of the span for increasing tension T_0.

B. Realistic situation

The mountain profile is shown in the upper part of Fig. 5 (a plant in South tyrol). For the optimization procedure we have choosen the mass of the point load as $m = 580$ kg, the cable type is as described in Fig. 3. The cable is held fixed at both ends of the line and the positions of three supports are determined by minimization of Eq. (11) subjected to constraints. The cable tension in the valley station is $T_0 = 173$ kN. The maximal cable tension during the ride is $T_{\rm max} = 192.6$ kN, occurring for $x_{\rm pl} \approx 700$ m. The constraint C_4 is fulfilled for $\nu = 3.1$.

V. SUMMARY AND OUTLOOK

In this work we have presented first results of our research on optimization of cable railways. We have defined a cost functional and constraints to find out optimal support positions by an optimization procedure, where we have analyzed local and global optimization strategies. To check the validity of our approach we have tested a simple linear mountain profile.

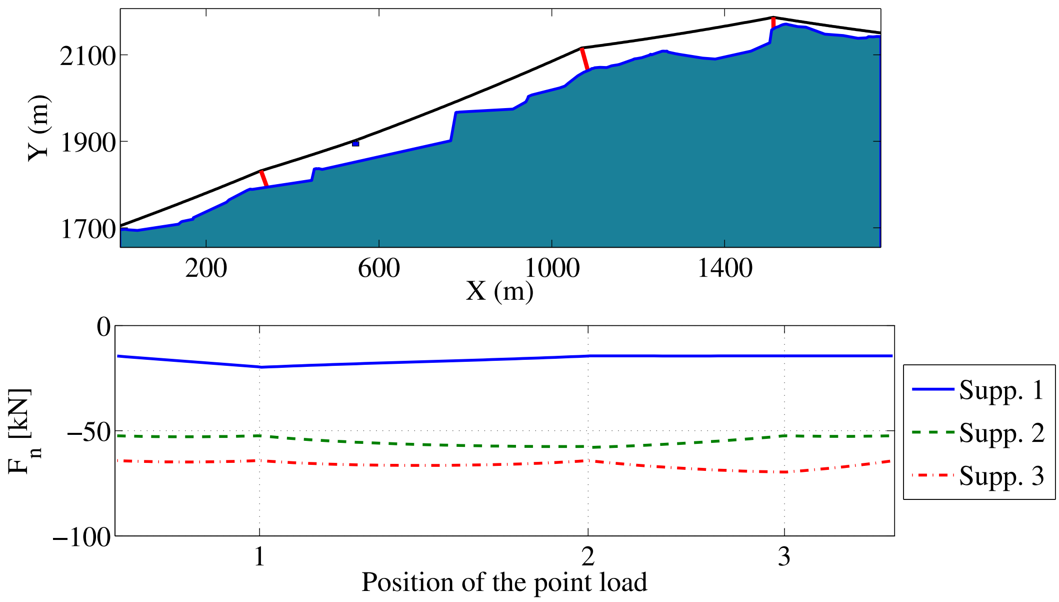

FIG. 5: The upper panel shows the mountain profile and the optimal solution found by local optimization. The lower figure shows the normal forces (the magnitude of the force times the sign of the $y-$ component) on the supports when the point load is moving. All forces are negative for all positions of the point load. The maximum value is about -15 kN and occurs on the support no. 1 when the point load is near the mountain station. For the constraints C_1, C_2, C_3 we have set $F^j_{y,\min} = -5$ kN for all j.

The results are reasonable and as expected from intuition. A second example, based on a real existing plant, shows reasonable results. The constraints regarding cable car–ground distance, support forces and allowed support cable tension are fulfilled. Further studies are needed to incorporate additional constraints and to ensure robustness of the solutions. Parallelization of the computations becomes essentially to reduce cpu–times.

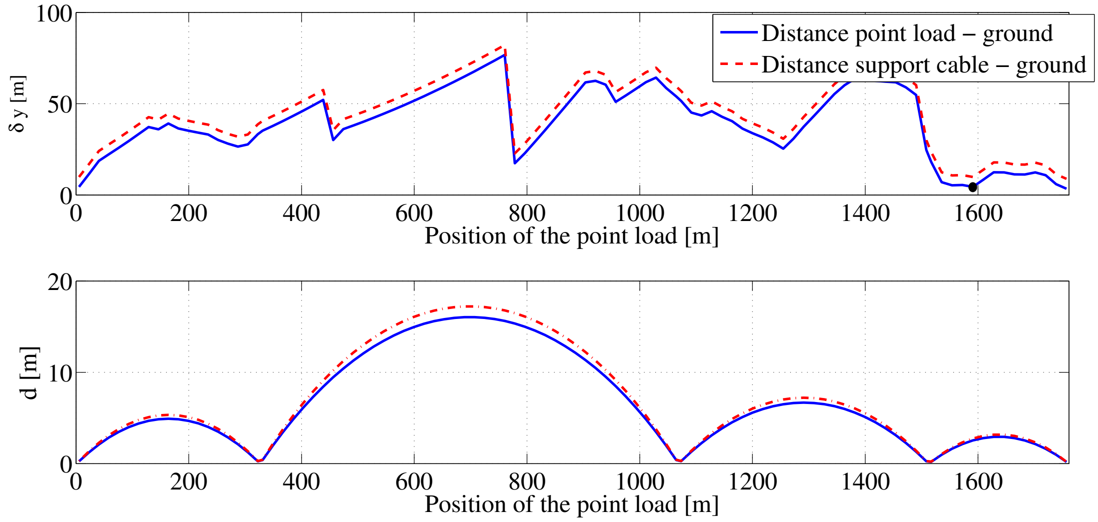

FIG. 6: The upper part shows the support cable – ground distance during the ride. The minimum is indicated by a black circle and is equal 5 m as required by the constraint C_0. The lower panel shows the distance d of the point load from the chord. The blue line corresponds to a certain reference temperature, for which the optimization was executed. To demonstrate the (important) effect of temperature we have also plotted the same distance for a temperature, which is 30 K higher. The maximum increase of d is about 1.2 m and occurs in the second span.

VI. ACKNOWLEDGEMENT

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[16] The point load is sitting on the middle of the span.

[17] See for example Mathematica Documentation Center, or the MATLAB Documentation.

[18] We exclude here the case of an overhanged rock formation.

[19] As usual $\mathbf{e}_x$, $\mathbf{e}_y$ are the canonical basis vectors.

Figure captions

Fig. 1

Planning of a cable railway: aspects of the planning steps.

Fig. 2

Mechanical model used in our numerical simulations.

Fig. 3

Results for a linear profile (slope 1/2) with 1 support. Two cases with different spanning weight T_0 are shown: For $T_0 = 100$ kN one obtains $J \equiv h_1 = 105.5$ m and for $T_0 = 200$ kN $J = h_1 = 55.6$ m ($\delta y_{min} = 5$ m). Dashed lines indicate the empty cables. Cable parameters here and in all other figures: a locked coil rope with diameter 21 mm and a metallic cross section of 301 mm^2 was used. The linear mass density is $\rho_L = 2.53$ kg/m, the breaking tension $T_{\rm br} = 530$ kN, the mass of the point load is $m = 1000$ kg respectively.

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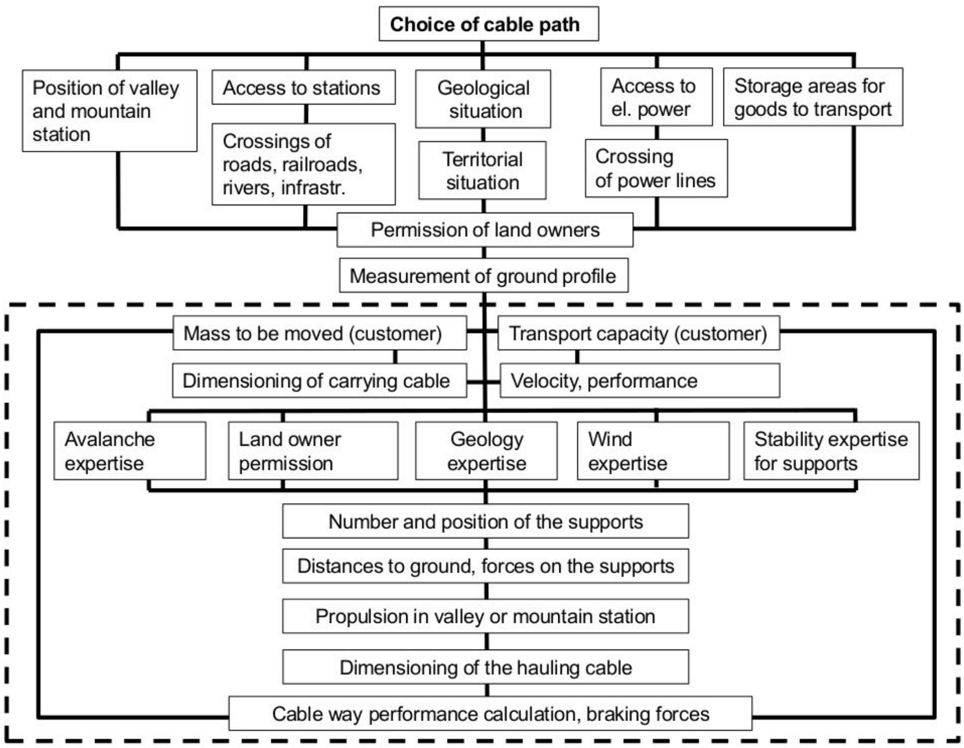
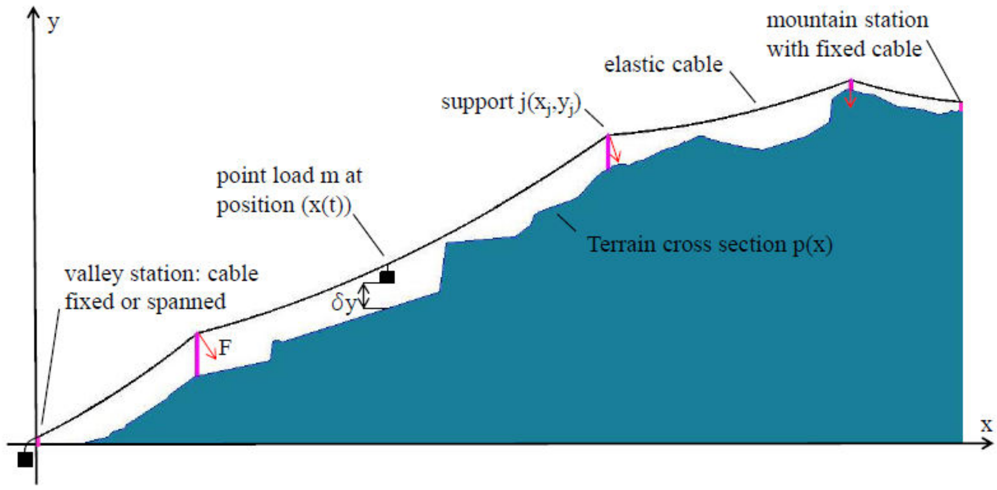
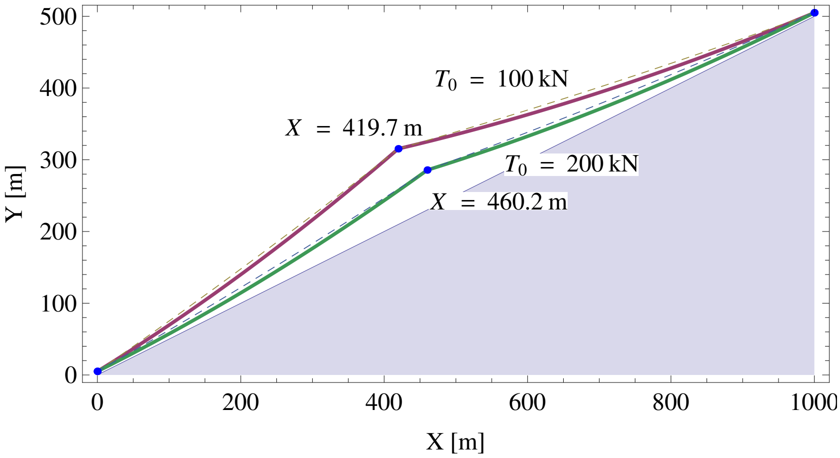
Table captions

Optimal support positions and heights for a cable fixed at the end points and spanned with a counterweight T_0. Parameters as in Fig. 3.


Choice of cable path
Position of valley and mountain station
Access to stations
Crossings of roads, railroads, rivers, infrastr.
Geological situation
Territorial situation
Access to el. power
Crossing of power lines
Storage areas for goods to transport
Permission of land owners
Measurement of ground profile
Mass to be moved (customer)
Transport capacity (customer)
Dimensioning of carrying cable
Velocity, performance
Avalanche expertise
Land owner permission
Geology expertise
Wind expertise
Stability expertise for supports
Number and position of the supports
Distances to ground, forces on the supports
Propulsion in valley or mountain station
Dimensioning of the hauling cable
Cable way performance calculation, braking forces


y
mountain station
with fixed cable
elastic cable
support j(x_j,y_j)
point load m at
position (x(t))
Terrain cross section p(x)
valley station: cable
fixed or spanned
δy
F
x



T_0 = 100 kN
X = 419.7 m
T_0 = 200 kN
X = 460.2 m
Y [m]
X [m]
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200
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0
200
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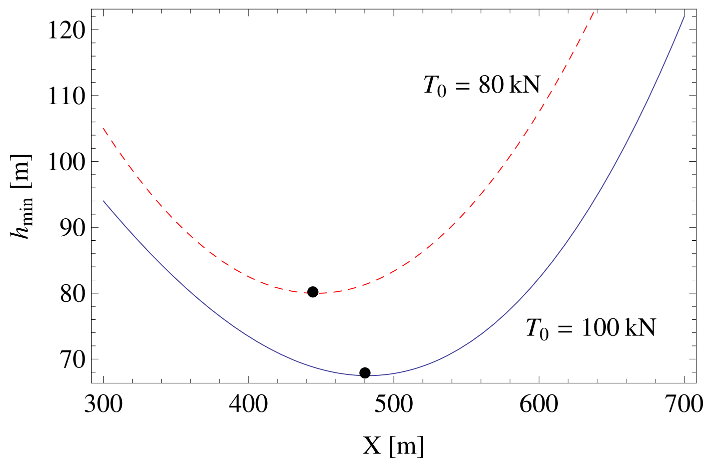
600
800
1000


120
110
100
90
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70
h_{min} [m]
300
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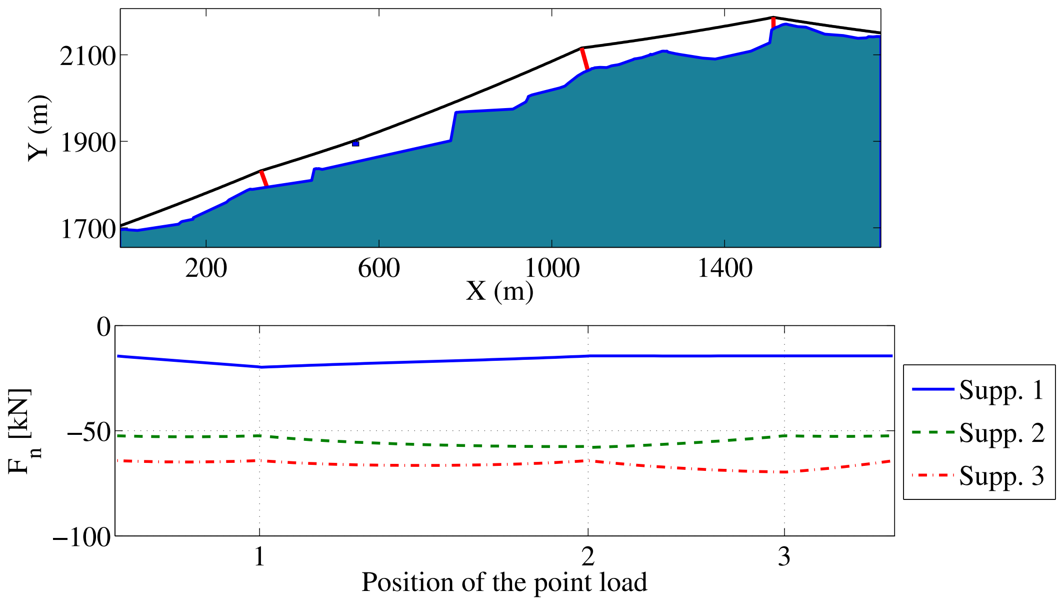
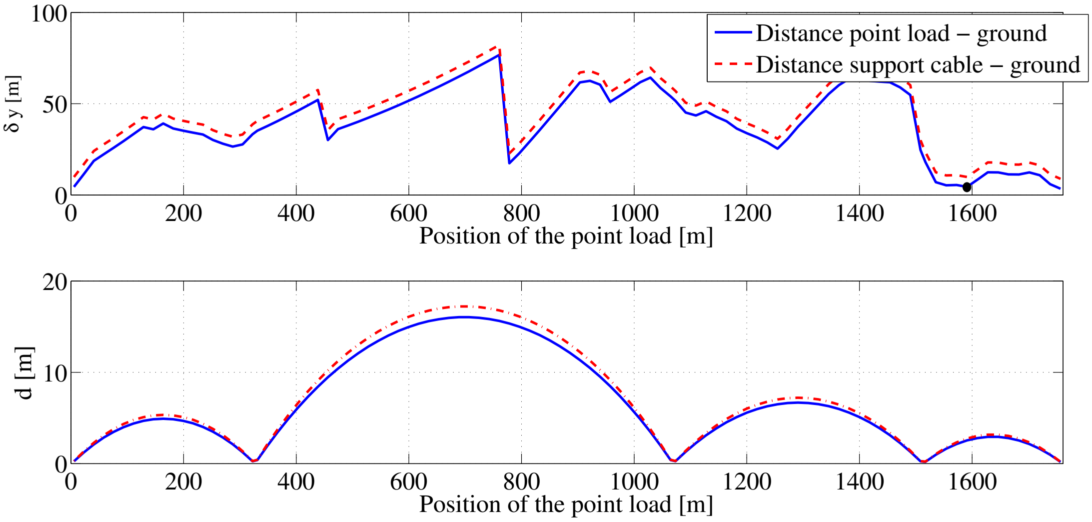
500
600
700
X [m]
T_0 = 80 kN
T_0 = 100 kN


2100
1900
1700
Y (m)
200
600
1000
1400
X (m)
0
−50
−100
F n [kN]
1
2
3
Position of the point load
Supp. 1
Supp. 2
Supp. 3


Distance point load – ground
Distance support cable – ground
δ y [m]
Position of the point load [m]
d [m]
Position of the point load [m]

bound. cond.	T_0 [kN]	100	150	200
end fix	X_{min} [m]	475	482.2	484.7
end fix	h_{min} [m]	63.3	49.1	40.3
sp. weight	X_{min} [m]	419.7	446.9	460.2
sp. weight	h_{min} [m]	105.5	72.2	55.6